NAG Toolbox for MATLAB

f07fa

1 Purpose

f07fa computes the solution to a real system of linear equations

$$AX = B,$$

where A is an n by n symmetric positive-definite matrix and X and B are n by r matrices.

2 Syntax

```
[a, b, info] = f07fa(uplo, a, b, 'n', n, 'nrhs_p', nrhs_p)
```

3 Description

f07fa uses the Cholesky decomposition to factor A as $A = U^{\mathrm{T}}U$ if **uplo** = 'U' or $A = LL^{\mathrm{T}}$ if **uplo** = 'L', where U is an upper triangular matrix and L is a lower triangular matrix. The factored form of A is then used to solve the system of equations $AX = B$.

4 References

Anderson E, Bai Z, Bischof C, Blackford S, Demmel J, Dongarra J J, Du Croz J J, Greenbaum A, Hammarling S, McKenney A and Sorensen D 1999 *LAPACK Users' Guide* (3rd Edition) SIAM, Philadelphia URL: http://www.netlib.org/lapack/lug

Golub G H and Van Loan C F 1996 *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

5.1 Compulsory Input Parameters

1: **uplo – string**

If **uplo** = 'U', the upper triangle of A is stored.

If **uplo** = 'L', the lower triangle of A is stored.

Constraint: **uplo** = 'U' or 'L'.

2: **a**(**lda,**$*$) **– double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

The n by n symmetric matrix A.

If **uplo** = 'U', the upper triangular part of A must be stored and the elements of the array below the diagonal are not referenced.

If **uplo** = 'L', the lower triangular part of A must be stored and the elements of the array above the diagonal are not referenced.

3: **b**(**ldb,**$*$) **– double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

The n by r right-hand side matrix B.

5.2 Optional Input Parameters

1: **n – int32 scalar**

Default: The second dimension of the array **a**.

n, the number of linear equations, i.e., the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

2: **nrhs_p – int32 scalar**

Default: The second dimension of the array **b**.

r, the number of right-hand sides, i.e., the number of columns of the matrix B.

Constraint: $\mathbf{nrhs\_p} \geq 0$.

5.3 Input Parameters Omitted from the MATLAB Interface

lda, ldb

5.4 Output Parameters

1: **a(lda,$*$) – double array**

The first dimension of the array **a** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{n})$

If $\mathbf{info} = 0$, the factor U or L from the Cholesky factorization $A = U^{\mathrm{T}}U$ or $A = LL^{\mathrm{T}}$.

2: **b(ldb,$*$) – double array**

The first dimension of the array **b** must be at least $\max(1, \mathbf{n})$

The second dimension of the array must be at least $\max(1, \mathbf{nrhs\_p})$

If $\mathbf{info} = 0$, the n by r solution matrix X.

3: **info – int32 scalar**

$\mathbf{info} = 0$ unless the function detects an error (see Section 6).

6 Error Indicators and Warnings

Errors or warnings detected by the function:

info $= -i$

If $\mathbf{info} = -i$, parameter i had an illegal value on entry. The parameters are numbered as follows:

1: **uplo**, 2: **n**, 3: **nrhs_p**, 4: **a**, 5: **lda**, 6: **b**, 7: **ldb**, 8: **info**.

It is possible that **info** refers to a parameter that is omitted from the MATLAB interface. This usually indicates that an error in one of the other input parameters has caused an incorrect value to be inferred.

info > 0

If $\mathbf{info} = i$, the leading minor of order i of A is not positive-definite, so the factorization could not be completed, and the solution has not been computed.

7 Accuracy

The computed solution for a single right-hand side, $\hat{x}$, satisfies an equation of the form

$$(A + E)\hat{x} = b,$$

where

$$\|E\|_1 = O(\epsilon)\|A\|_1$$

and ϵ is the ***machine precision***. An approximate error bound for the computed solution is given by

$$\frac{\|\hat{x} - x\|_1}{\|x\|_1} \leq \kappa(A)\frac{\|E\|_1}{\|A\|_1},$$

where $\kappa(A) = \left\|A^{-1}\right\|_1 \|A\|_1$, the condition number of A with respect to the solution of the linear equations. See Section 4.4 of Anderson *et al.* 1999 for further details.

f07fb is a comprehensive LAPACK driver that returns forward and backward error bounds and an estimate of the condition number. Alternatively, f04bd solves $Ax = b$ and returns a forward error bound and condition estimate. f04bd calls f07fa to solve the equations.

8 Further Comments

The total number of floating-point operations is approximately $\frac{1}{3}n^3 + 2n^2r$, where r is the number of right-hand sides.

The complex analogue of this function is f07fn.

9 Example

```
uplo = 'Upper';
a = [4.16, -3.12, 0.56, -0.1;
     0, 5.03, -0.83, 1.18;
     0, 0, 0.76, 0.34;
     0, 0, 0, 1.18];
b = [8.699999999999999;
     -13.35;
     1.89;
     -4.14];
[aOut, bOut, info] = f07fa(uplo, a, b)
```

```
aOut =
    2.0396   -1.5297    0.2746   -0.0490
         0    1.6401   -0.2500    0.6737
         0         0    0.7887    0.6617
         0         0         0    0.5347
bOut =
    1.0000
   -1.0000
    2.0000
   -3.0000
info =
           0
```